Implicit Level Set Methods for a Fire Spread Model

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Abstract

The level set method is a computational technique for tracking a moving interface over time. It can naturally handle topological changes such as merging or breaking interface. Intrinsic geometric properties of the interface, such as curvature and normal direction, are easily determined from the level set function. There are many applications of the level set method, including kinetic crystal growth, epitaxial growth of thin films, image restoration, vortex dominated flows etc. However, most applications advance the level set equation with explicit time integration. Hence, small time steps are needed to maintain stability. In this paper, an implicit level set method is introduced and applied to wildland fire spread model, removing vulnerability to instability.

KEY WORDS: Level set method, fire spread model, Newton method, Newton-GMRES.

Introduction

Level set methods, introduced by Osher and Sethian (1988), are numerical techniques designed for tracking the evolution of moving interfaces. Marker methods and Volume-of-fluid methods can also be used to track moving interfaces.

Markers methods involve planting marker particles along the propagating interfaces and follow their movement. This Lagrangian technique can be practical for calculating small perturbative motions. However, for large and complex deformations, systematic methods for removing or adding marker points as they get too close together or too far apart are required, and complex data structures are required to accommodate topological change, such as pinch-off or merge, and to prevent tangling of the interface in regions of high curvature.

Volume-of-fluid methods embed the interface in an Eulerian grid in which it is tracked implicitly. We lay down a fixed grid on the computational domain and assign a value to each grid cell based on the fraction of the interior currently located in that cell. An advantage of the Volume-of-fluid technique is that topological complications boundaries are accommodated without effort. However, calculation of intrinsic geometric properties of the interface, such as curvature and normal direction, is difficult and inaccurate.

Level set methods are well known for their ability to easily handle topological changes such as merging or breaking interfaces. The position of the interface is defined by the zero level set. Intrinsic geometric properties of the interface are easily calculated from the level set function. Level set methods are relatively easy to implement. They build on the established theory of Hamilton-Jacobi equations. Thus, we can exploit techniques borrowed from the numerical solution of Hamilton-Jacobi equations.

This paper focuses on the application of level set methods to wildland fire spread model. Understanding and control of wildfire is a high priority in contemporary forest management. A century of active fire suppression has led to ecosystems out of balance with respect to fire. Fire modeling has been proposed to predict the speed of wildland fires in order to help containment and management of resource during a fire incident, as well as to plan controlled burns.

In 1982, Anderson *et al.* (1982) proposed a commonly used fire spread model with the long axis stretched out in the wind direction in which the fire appears as an elliptical front. However, such models are inaccurate for complicated fire spread in practice. Other first-principles approaches use conservation of mass, momentum, and energy as a tool for modeling and predicting the speed of firefront. However, it is very difficult to measure and calculate such properties and to accurately model a three dimensional geometry of forests in large-scale of wildfire. This is a classic multiscale problem for which routine simulation is not possible. In our application, we use an semi-empirical fire spread model proposed by Fendell and Wolff (see Fendell and Wolff, 1991) that requires a minimal amount of information about properties of the environment around the firefront.

Experience with explicit level set implementations of these fire spread models shows numerical instability unless the timestep is carefully chosen. Therefore, in this research, we want to implicitly solve the level set equation. Hence, we need to choose an efficient implicit nonlinear solver. Newton's method is a leading candidate for solving a nonlinear system of equations of the form $F(x) = 0$. It is attractive because of the asymptotically quadratic rate of convergence. However, computing a Newton step can be expensive when the size of the problem is large. Instead of exactly computing a Newton step, Dembo *et al*. (see Dembo *et al.,* 1982) introduces an inexact Newton method in which an iterative method is used to compute a Newton step approximately. Both Newton's method and inexact Newton methods require explicitly formation of a complete Jacobian matrix. For very large problems, it can be expensive to store a Jacobian matrix. However, we can use Newton-GMRES method to overcome the storage problem. The Newton-GMRES method requires only products between a Jacobian matrix and a vector. These can be approximated by using finite differences. Hence, the Newton-GMRES can be implemented without explicitly forming the Jacobian matrix.

Level set methods

The level set methods are a computational method for tracking interfaces. At any time *t*, the interface is defined by the zero level set of the smooth time-dependent function $\phi(x,t)$. Hence, the interface is the set $\{x \mid \phi(x,t) = 0\}$. Then, by chain rule, we have an evolution equation for the interface

$$\phi_t + F \mid \nabla\phi \mid = 0, \qquad [1]$$

where *F* is a speed function. The level set methods have several advantages. First, although the level set function remain well defined, the level set surface may change topology. The interface may merge or break. Second, intrinsic geometric properties of the interface are easily to perform. For example, the normal direction to the interface is given by $\nabla\phi / \mid \nabla\phi \mid$ and the curvature of the interface is given by $\nabla \cdot \left(\nabla\phi / \mid \nabla\phi \mid\right)$. Third, the formula is generalized in their coordinate invariant form to a propagation in higher dimension. Finally, the level set methods are easily to implement. They can be linked to hyperbolic conservation laws. Hence, we can use techniques borrowed from numerical solution of hyperbolic conservation laws.

Fire spread model

Fendell and Wolff (1991) use experiments in their test facility to propose a semi-empirical fire spread model. This model specifies the speeds of the firefront at head, flank and rear, and elsewhere trigonometrically interpolates between them (Figure 1).

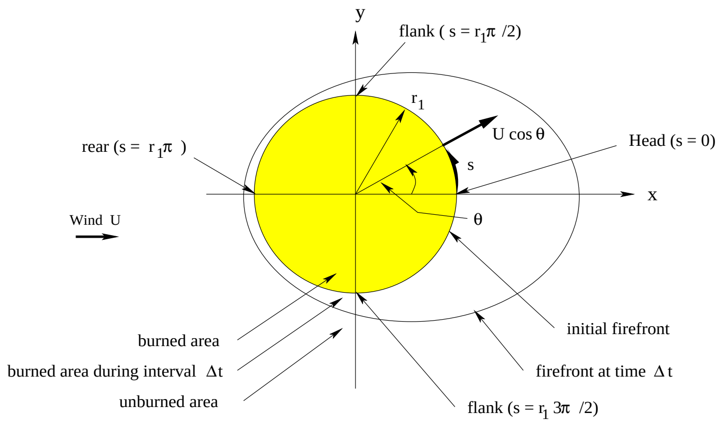

Figure 1: A circular initial burned area with radius r_1. The wind of magnitude U in the direction θ = 0 results the fire to grow into an elliptical shape during the time interval Δ *t*.

The model concentrates on wind effects on the fire interface. Fendell and Wolff (1991) suggest that the local speed for a firefront, based on the wind speed and direction with respect to the local front, is given by ($\mu \approx$ 2-4)

$$F(U,\theta) = \begin{cases} v_f(U\cos\theta) + \beta(U\sin^\mu\theta), |\theta| < \frac{\pi}{2} \\ \beta(U\sin^\mu\theta) + \varepsilon(U\cos^2\theta), |\theta| > \frac{\pi}{2} \end{cases} \quad [2]$$

where U denotes the wind speed, θ denotes the angle (measured counterclockwise) between front normal and wind direction, μ denotes the exponent for shape control, and v_f, β and ε are directional modifiers at the head, flanks and rear, respectively. Fendell and Wolff propose that the wind speed behavior at the head, flanks and rear are $v_f(U) = \varepsilon_0 + c_1\sqrt{U}$, $\beta(U) = \varepsilon_0 + aU\exp(-bU)$ and $\varepsilon(U) = \varepsilon_0\exp(-\varepsilon_1 U)$, respectively. Here, c_1, ε_0, ε_1, a and b are parameters independent of the wind speed U and angle θ.

Newton-GMRES method

Newton-GMRES use GMRES method proposed by Saad and Schultz (see Saad and Schultz, 1986) to find solutions of the linear system

$$J(u^{k-1})\delta u^k = F(u^{k-1}), \quad [3]$$

where J is the Jacobian matrix and $F(u)$ is a nonlinear system of equation. GMRES requires only products between a matrix and a vector. These can be approximated by finite differences.

The product between the Jacobian matrix $J(u)$ and a vector δu can be approximated by

$$J(u)\delta u \approx \frac{F(u+h\delta u)-F(u)}{h}, \quad [4]$$

where h is a small perturbation scalar. A simple choice (Knoll and Keyes, 2004) is

$$h \approx \frac{\sqrt{(1+\|u\|)\varepsilon_{mach}}}{\|\delta u\|}, \quad [5]$$

where ε_{mach} is the value of machine epsilon. Hence, Newton-GMRES methods will have Newton-like nonlinear convergence without forming or storing the true Jacobian.

Numerical approximation

In 1988, Osher and Sethian (see Osher and Sethian, 1988) proposed a numerical approximation of $|\nabla\phi|$ where $|\nabla\phi| = \sqrt{\phi_x^2 + \phi_y^2}$.

If the speed function $F > 0$ then the approximation of $|\nabla\phi|$ is given by

$$\nabla^{+} = \left(\max(D^{-x}\phi_{ij},0)^2 + \min(D^{+x}\phi_{ij},0)^2 + \max(D^{-y}\phi_{ij},0)^2 + \min(D^{+y}\phi_{ij},0)^2\right)^{\frac{1}{2}} . \quad [6]$$

If the speed function $F < 0$ then the approximation of $|\nabla\phi|$ is given by

$$\nabla^{-} = \left(\min(D^{-x}\phi_{ij},0)^2 + \max(D^{+x}\phi_{ij},0)^2 + \min(D^{-y}\phi_{ij},0)^2 + \max(D^{+y}\phi_{ij},0)^2\right)^{\frac{1}{2}} . \quad [7]$$

However, if we do not know the sign of the speed function *F* ahead of time then the product between the speed function *F* and $|\nabla\phi|$ is approximated by

$$F|\nabla\phi| \approx \left(\max(F_{ij},0)\nabla^{+} + \min(F_{ij},0)\nabla^{-}\right) \quad [8]$$

Finally, the implicit numerical scheme of a level set equation used in this research is given by

$$\phi_{ij}^{n+1} - \phi_{ij}^{n} + \Delta t\left(\max(F_{ij}^{n+1},0)\nabla^{(n+1)+} + \min(F_{ij}^{n+1},0)\nabla^{(n+1)-}\right) = 0 , \quad [9]$$

where ϕ_{ij}^{n} is the value of ϕ at point (x_i, y_j) at time t_n. If an explicit numerical scheme is used, then we need to enforce numerical stability by using the Courant-Friedrichs-Lewy (CFL) condition. Since the propagation speed of numerical waves should be at least as fast as the speed of the physical waves, we have $\Delta x/\Delta t > |F|$. Hence, the CFL condition is $\Delta t < \Delta x/\max(|F|)$ where $\max(|F|)$ is chosen to be the largest value of *F* over the entire computational domain.

Numerical results

We perform numerical experiments on our fire spread model. We use Fendell-Wolff model with $\mu = 1.5$ in our experiments. The code written using PETSc software library is run parallel on a SUN enterprise 2900 server. We use matrix free Newton-GMRES method as a nonlinear solver and use PETSc's default convergence criterion in the experiments. We choose computation domain [0,3]x[0,3], the spatial step $\Delta x = \Delta y = 0.01$ and then choose the time step $\Delta t = 0.02$ violating the CFL condition. Since data for realistic fire is not available at the time of writing, we arbitrarily choose fire parameters as followed, $U = 5.0$, $a = 0.1$, $b = 1.0$, $\varepsilon_0 = 0.1$, $\varepsilon_1 = 0.003$, and $c_1 = 0.5$. For comparison purposes, each Figure shows superimposed front positions at regularly spaced times.

We begin our experiment with a circle of radius 0.5 centered at (1, 1.5). We allow a wind constantly blows from the west to the east. The result of experiment is shown in Figure 2. The fire advance at the head much faster than at the flank and rear as intended.

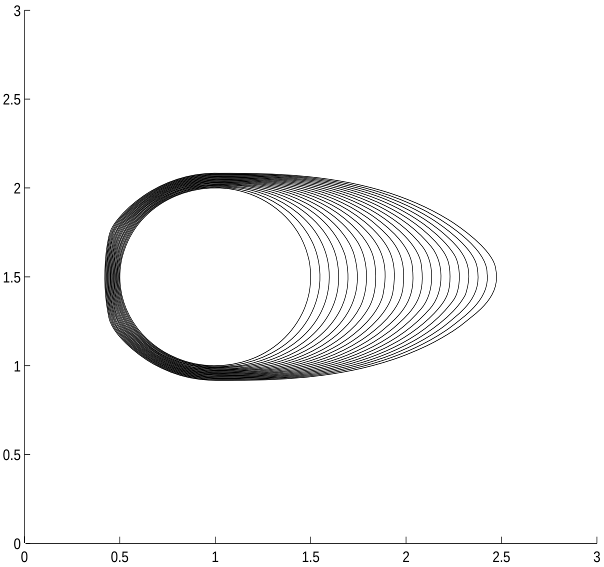

Figure 2: Fire propagation under a steady wind.

Next, we perform an experiment when a wind blows from the west and slowly turns to acquire a southerly component. The result is shown in Figure 3. The fire initially advances to the east and then gradually advances to the north-east.

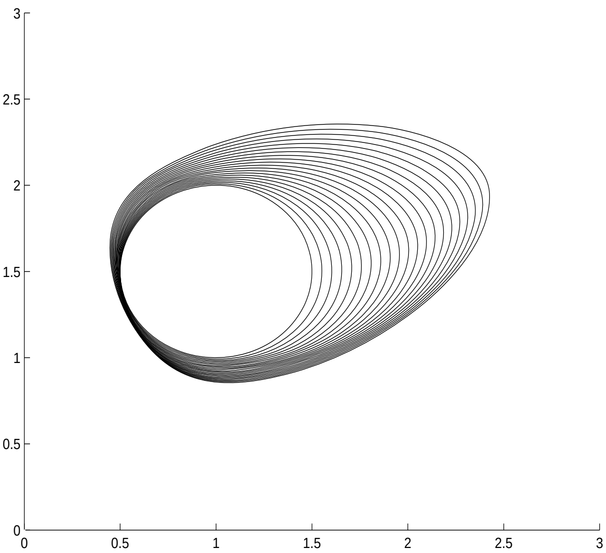

Figure 3: Fire propagation under a turning wind.

Next, we perform an experiment in which we have three initial curves. The first curve is a circle of radius 0.5 centered at (1, 1), the second is a circle of radius 0.3 centered at (1.5, 1.8), and the third is a circle of radius 0.25 centered at (1, 1). Here, the third curve demarcates an unburned island inside of a burning region. The result is shown in Figure 4.We observe the unburned island gradually disappearing and the two fire fronts eventually merging into a single fire.

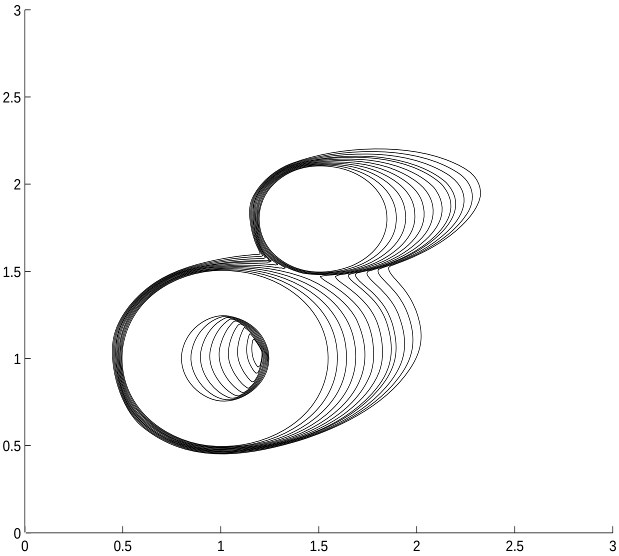

Figure 4: Fire propagation of two elliptical fires, one with an unburned area. The fires evolve under a wind from the west and gradually turn to be from the south.

Finally, we perform an experiment for which the energy density of the fuel varies across an east-west line. We begin with a circular fire of radius 0.4 centered at (1.5, 1.5). We set the region above the line $y = 2$ to have a low energy density while the region below the line $y = 2$ has high density. In this test, there is no wind. The result is shown in Figure 4. As intended, the fire advances more quickly within the high energy density region. Once the fire reaches the low density region, it advances more slowly.

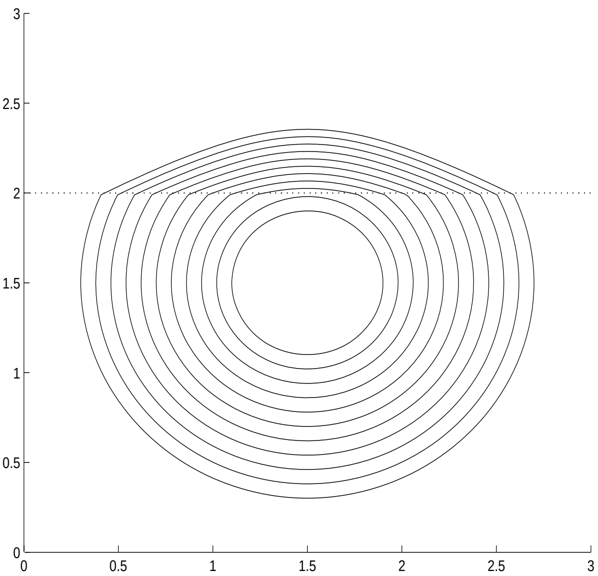

Figure 5: An elliptical fire propagates without wind from a region of high fuel density into a region of low fuel density.

Conclusions and future works

The main goal of this research is to introduce an implicit new level set method for wildland fire spread. An advantage of an implicit method is that it is possible to use a time step of arbitrary size, determined only by accuracy criteria and free of instability. Our numerical experiments show that the implicit level set method works well with a large time step.

Although our implicit level set method works well, several improvements can be made to the method to make it more efficient. In this research, we have used a full matrix approach of the level set method where the value of ϕ is calculated at every points on the computational domain. This can be expensive. A narrow band approach in which only computational grids close to the zero level set are used would be worthwhile to implement. This would require dynamic data structures, but would significantly reduce the computational complexity.

References

Anderson, D.H., Catchpole, E.A., De Mestre, N.J., and Parkes, T. 1982. Modeling the spread of grass fires. Australian Mathematical Society (Series B) 23:451-466.

Dembo, R.S., Eisenstat, S.C., and Steihaug, T. 1982. Inexact Newton methods. SIAM Journal on Numerical Analysis 19:400-408.

Fendell, F.E., and Wolff, M.F. 1991. Wind-Aided Fire Spread. p. 171-223. In: Johnson, E.A., and Miyanishi, K. (Eds.). Forest Fires: Behavior and Ecological Effects. New York: Academic Press.

Knoll, D.A., and Keyes, D.E. 2004. Jacobian-free Newton-Krylov methods: a survey of approaches and applications. Computational Physics 193:357-397.

Osher, S., and Sethian, J.A. 1988. Fronts propagating with curvature dependent speed: Algorithms based on Hamilton-Jacobi formulations. Computational Physics 79:12-49.

Saad, Y., and Schultz, M.H. 1986. GMRES: A Generalized minimal residual algorithm for solving nonsymmetric linear systems. SIAM Journal on Scientific and Statistical Computing 7:856-869.

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